NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

THESIS

**TARGET MOTION ANALYSIS
FROM A
DIESEL SUBMARINE'S PERSPECTIVE**

by

Pedro F. Coll

September, 1994

Thesis Advisor: Alan R. Washburn

Approved for public release; distribution is unlimited.

REPORT DOCUMENTATION PAGE	Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE September 1994	3. REPORT TYPE AND DATES COVERED Master's Thesis

4. TITLE AND SUBTITLE TARGET MOTION ANALYSIS FROM A DIESEL SUBMARINE'S PERSPECTIVE	5. FUNDING NUMBERS
6. AUTHOR(S) Pedro F. Coll	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey CA 93943-5000	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.	12b. DISTRIBUTION CODE A

13. ABSTRACT *(maximum 200 words)*

The subject of this thesis is diesel submarine tactics for performing an effective Bearing-only Target Motion Analysis while approaching a surface target, given the submarine's limited speed and battery endurance. The research is conducted through a Monte Carlo simulation. Each replication simulates a submarine's approach, attack and eventual torpedo release, to determine the success or failure of the attack. The number of successful attacks in every replication is a measure of effectiveness of the particular tactics employed. The simulation shows that a modern diesel submarine is capable of conducting a Bearing-only TMA while at the same time approaching a surface target.

14. SUBJECT TERMS TMA, Target Motion Analysis, Diesel Submarine.	15. NUMBER OF PAGES 54
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89)

Prescribed by ANSI Std. 239-18 298-102

Approved for public release; distribution is unlimited.

TARGET MOTION ANALYSIS
FROM A
DIESEL SUBMARINE'S PERSPECTIVE

Accesion For		
NTIS CRA&I		X
DTIC TAB		☐
Unannounced		☐
Justification		
By		
Distribution /		
Availability Codes		
Dist	Avail and / or Special	
A-1		

by

Pedro F. Coll
Lieutenant Commander, Spanish Navy
Spanish Naval Academy, 1975

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1994

Author: Pedro F. Coll

Approved by: Alan R. Washburn, Thesis Advisor

James N. Eagle, Second Reader

Peter Purdue, Chairman
Department of Operations Research

ABSTRACT

The subject of this thesis is diesel submarine tactics for performing an effective Bearing-only Target Motion Analysis while approaching a surface target, given the submarine's limited speed and battery endurance. The research is conducted through a Monte Carlo simulation. Each replication simulates a submarine's approach, attack and eventual torpedo release, to determine the success or failure of the attack. The number of successful attacks in every replication is a measure of effectiveness of the particular tactics employed. The simulation shows that a modern diesel submarine is capable of conducting a Bearing-only TMA while at the same time approaching a surface target.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

EXECUTIVE SUMMARY

The purpose of this thesis is to investigate the possibilities that a diesel submarine has to perform an effective Target Motion Analysis (TMA), while approaching a surface target, given its limited speed and battery endurance, and to provide an evaluation of the best tactics to be used by the submarine.

The submarine is assumed to be conducting a barrier patrol against a surface target. The target will be a ship, heading at constant course and speed, which has been detected by the submarine using its hull-mounted hydrophone array. Assuming that hostile ASW units may be present in the area, the submarine will conduct the approach and attack submerged, avoiding the use of the periscope or any active sensor, so a Bearing-only TMA will be needed to compute firing data.

The research is conducted through a Monte Carlo simulation.

The main elements of the simulation program are:

- Submarine model, including motion characteristics, battery, sensor and weapons performances.
- Target model, including motion characteristics.
- Implementation of TMA procedures: Expanded Time-Bearing Plot, Time-corrected Ekelund Range, Relative Motion Plot and Geographic Plot.

- Implementation of submarine tactics: a total of 9 tactical variations are evaluated.

Each replication simulates a submarine's approach, attack and eventual torpedo release, to determine the success or failure of the attack. Each simulation includes 10,000 replications and is run with parameters that depend on the tactics used.

The simulation shows that a modern diesel submarine is capable of reaching a favorable attack position, closing the target within torpedo range and generating a TMA accurate enough to place the weapon within acquisition range, even taking into account its battery limitations.

I INTRODUCTION

For about forty years NATO Navies were focussed against the maritime threat, predominantly submarine, of the Soviet Union and Warsaw Pact. The main role assigned to NATO diesel submarines in case of conflict during that period would have been the conduct of antisubmarine warfare (ASW) operations, mostly in restricted environments like the Baltic Sea, Gibraltar Strait, etc., where a diesel boat may be more cost/effective than a nuclear submarine.

Current defense policies would shift the Armed Forces' objective to deal with Low Intensity or Regional Conflicts. For NATO Navies that means a new role as Peace enforcement forces, giving a new emphasis to the Mediterranean Sea, where that kind of conflict is more likely to happen. The mission for NATO diesel submarines in that event will be the enforcement of a blockade or the interdiction of enemy shipping and/or naval operations in an area of exclusion. Examples are the operations conducted by British submarines during the Falklands conflict.

The subject of this thesis is one aspect of that mission: the Target Motion Analysis (TMA) that a modern diesel submarine performs, while approaching a surface target, given the submarine's limited speed and battery endurance.

A. PROBLEM DEFINITION

The submarine is assumed to be conducting a barrier patrol against surface targets. The target will be a ship, with a constant course and speed, which has been detected by the submarine using its hull-mounted hydrophone array. Assuming that hostile ASW units may be present in the area, the submarine will conduct the approach and attack submerged. This avoids the use of the periscope or any active sensor, and requires that Bearing-only TMA will be used in order to compute firing data.

When a submarine commander conducts an approach, he is trying to achieve the following objectives:

1. Close the target within torpedo range.
2. Remain undetected at least until torpedo release.
3. Generate a TMA accurate enough to place the weapon within its target acquisition.

Each of these objectives imposes tactical restrictions on the submarine. Ideally, the Commanding Officer wants to complete all three objectives, but tactical or technical constraints may not always allow it, especially in the case of a diesel boat.

A favorable attack position can be reached if all three objectives are met, but in order to do this the submarine must maneuver. A complete Bearing-only TMA cannot be performed if the submarine remains in a steady course and speed. On the

other hand, the submarine may be unable to close the target within weapon range if it maneuvers too much. There is a trade off between objectives one and three: the TMA maneuver may provide the required accuracy, but at the same time prohibit closure. That problem is particularly severe in the case of a diesel boat because of battery limitations.

B. METHODOLOGY

The research is conducted through a Monte Carlo simulation, the object being to determine the conditions under which a diesel submarine is able to perform an effective TMA while approaching a given surface target, and to provide an evaluation of the best tactics to be used by the submarine.

In order to do that, each replication simulates a submarine's approach, attack and eventual torpedo release, to determine the success or failure of the attack. Each simulation includes 10,000 replications and is run with different parameters that depend on the tactics used in each case; the number of successful attacks in every replication is a measure of effectiveness (MOE) of the particular tactics employed.

II TARGET MOTION ANALYSIS PROCEDURES

Bearing-only TMA techniques use the available information (Bearing, Bearing-rate and Time), to compute target data (Range, Course and Speed), without making any assumption other than that the target is heading at a constant course and speed. The following sections describe several of these techniques that are in common use among NATO Submarine Forces, together with how these procedures are implemented in the simulation. The simulation in all other respects is described in Chapter III.

Sonar bearings received from the hydrophone array contain random errors (assumed normally distributed, with mean zero and a given standard deviation) that must be smoothed for TMA. For manual plotting techniques, the sonar bearings are smoothed by fitting a curve or straight line through the raw bearings plotted on the Expanded Time-Bearing Plot, Figure I, a two-dimensional grid on which the horizontal scale represents bearing and the vertical scale represents time. The fitted curve provides a more accurate bearing that can be read from the curve at any point in time. The slope of this curve represents the bearing-rate. This plot provides all the data that are used as inputs by subsequent TMA procedures.

Figure I represents a portion of the plot that includes two legs. A leg is a period of time on which the submarine

maintains constant course and speed. *LEG 1* lasts from minute 11 to 15 and data obtained for this leg are mean time *t1*, mean bearing *B1* and bearing rate *Br1* (degrees per minute). *LEG 2* lasts from minute 17.5 to 22. The corresponding data are mean time *t2*, mean bearing *B2* and bearing rate *Br2*; both legs are separated by a change of course and speed to 200 degrees and 10 knots respectively.

Each iteration of the program main loop corresponds to 30 seconds of real time. For each iteration, submarine and target current positions are actualized, according to the respective

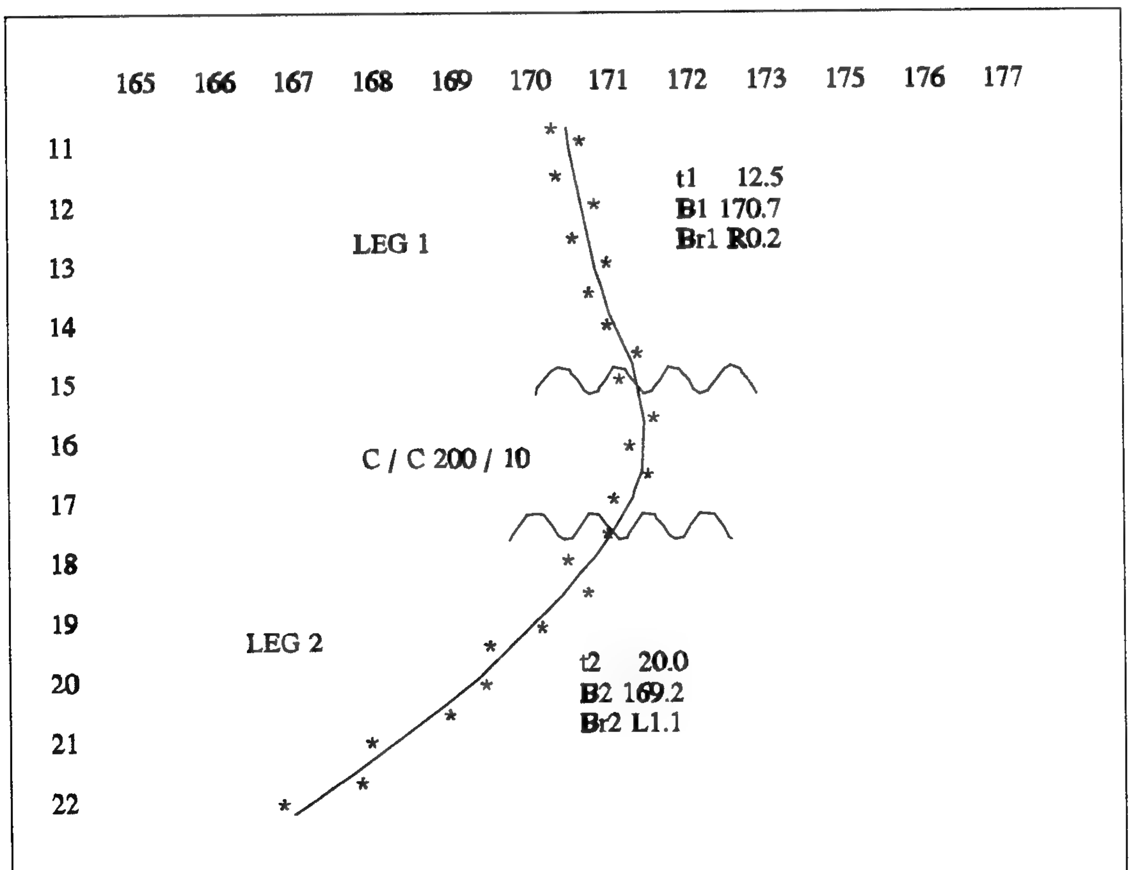

Figure I. Expanded Time-Bearing Plot

courses and speeds. Target actual bearing is computed from these data and received bearing is obtained by adding the measurement error, distributed Normal(Mean = 0.0, Standard Deviation = 0.5 degrees). Both time and received bearing are stored in linear arrays,with each pair (*time[i], bearing[i]*) representing a point in the plot. Mean time, mean bearing and bearing rate are computed by least squares regression and stored in two-dimensional arrays, indexed by leg number and leg part (first or second half of the leg).

A. TIME-CORRECTED EKELUND RANGE

1. Description

Ekelund ranging, perhaps the most extensively used passive ranging method, consists in two legs separated by a turn. Mean times *t1*, *t2*, mean bearings *B1*, *B2*, bearing-rates *Brate1*, *Brate2* and the components of submarine speed across the line of sight *SSalos1*, *SSalos2* are computed for each leg. Ekelund range is obtained by dividing the difference between submarine speeds across the line of sight by the difference between bearing-rates:

$$Rek=\frac{SSalos2-SSalos1}{Brate1-Brate2} \tag{1}$$

Where *Rek* is the Ekelund range. This formula was originally derived by J. J. Ekelund, reference [1]. The complete passive ranging equation involves more parameters, as is described in Chapter III, Section 3.2, of reference [2], but

this approximation is used in the simulation and in reality as well.

In Ekelund's derivation, range was assumed constant over the ranging maneuver, but in practice range depends on time, so it is necessary to find when the *Rek* estimate is in some sense most accurate. The Time Correction Procedure is employed for this purpose, usually obtaining the best range time, *Tstar,* by the graphical method described in Chapter III, Section 3.2 and Figure III-2, of reference [2]. The procedure yields:

$$Tstar=\frac{t1*Brate1-t2*Brate2+B1-B2}{Brate1-Brate2} \tag{2}$$

Figure II shows an example of this graphical method.

2. Implementation

In the simulation, when a range maneuver is completed, mean time, mean bearing and bearing rate are obtained for each leg, from the time-bearing plot, using the five minutes (11 data points) immediately before and after the turn. Since a complete leg is composed of 22 data points, this corresponds to half a leg.

In addition, submarine speeds across the line of sight are computed for each leg, at times corresponding to both mean bearings.

The Ekelund range and the best time for this range, *Tstar*, are obtained by plugging those data into the Ekelund

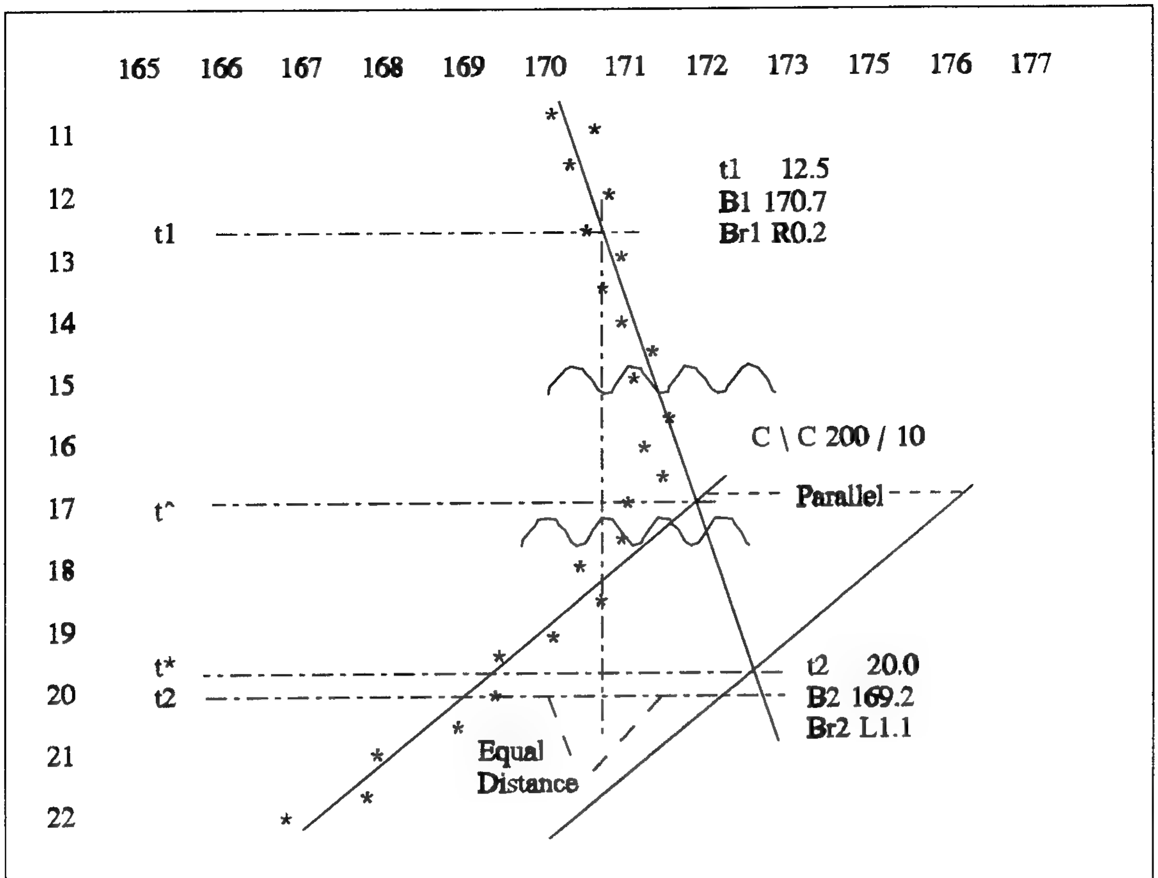

Figure II. Construction for the Best Time Range

and time-corrected equations.

B. RELATIVE MOTION PLOT

1. Description

The Relative Motion Plot is based on a geometric property of the relative movement that results when submarine and target have constant courses and speeds. If bearings *B1*, *B2*, *B3* are taken from submarine to target, at times *t1*, *t2*, *t3*, then from Figure III, applying the law of sines:

$$R3*\sin(B3-B2)=TgtS*(t3-t2)*\sin[RelBeta-(B3-B2)] \quad (3)$$

and

$$R3*\sin(B3-B1)=TgtRelS*(t3-t1)*\sin[RelBeta-(B3-B1)] \quad (4)$$

Dividing (3) by (4),

$$\frac{\sin(B3-B2)}{\sin(B3-B1)}=\frac{(t3-t2)*\sin[RelBeta-(B3-B2)]}{(t3-t1)*\sin[RelBeta-(B3-B1)]} \quad (5)$$

Rearranging,

$$\frac{(t3-t1)}{(t3-t2)}=\frac{\sin(B3-B1)\sin[RelBeta-(B3-B2)]}{\sin(B3-B2)\sin[RelBeta-(B3-B1)]} \quad (6)$$

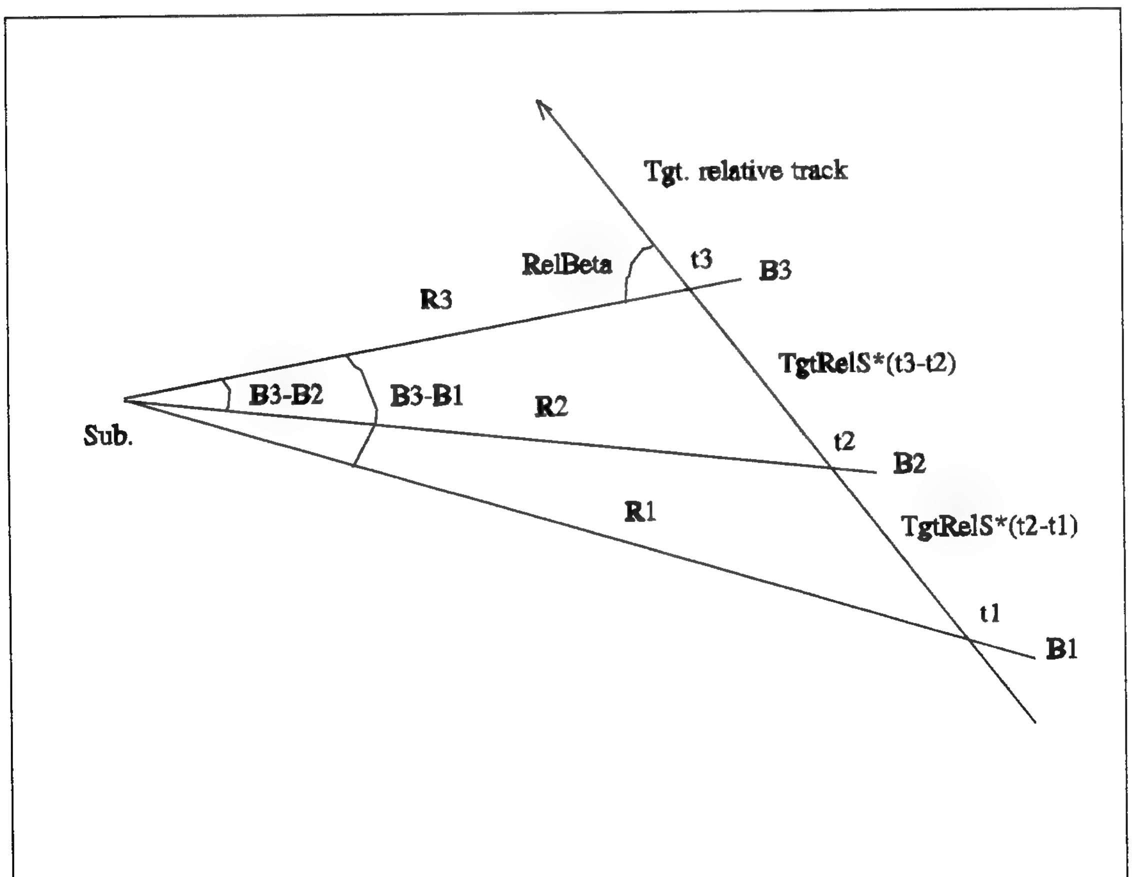

Figure III. Derivation of RelBeta

Now applying the formula for the sine of difference to numerator and denominator and simplifying,

$$\frac{(t3-t1)}{(t3-t2)}=\frac{\sin(RelBeta)\cot(B3-B2)-\cos(RelBeta)}{\sin(RleBeta)\cot(B3-B1)-\cos(RelBeta)} \tag{7}$$

Dividing numerator and denominator by *sin(RelBeta)*,

$$\frac{(t3-t1)}{(t3-t2)}=\frac{\cot(B3-B2)-\cot(RelBeta)}{\cot(B3-B1)-\cot(RelBeta)} \tag{8}$$

Then rearranging,

$$\frac{(t3-t1)}{\tan(B3-B1)}-\frac{(t3-t1)}{\tan(RelBeta)}=\frac{(t3-t2)}{\tan(B3-B2)}-\frac{(t3-t2)}{\tan(RelBeta)} \tag{9}$$

and grouping terms,

$$\frac{(t2-t1)}{\tan(RelBeta)}=\frac{(t3-t1)}{\tan(B3-B1)}-\frac{(t3-t2)}{\tan(B3-B2)} \tag{10}$$

Here *RelBeta* is the angle between the direction of the relative movement and the third bearing. If the three bearings are taken at constant time intervals, as is represented in Figure III, then equation (10) becomes:

$$\cot(RelBeta)=2*\cot(B3-B1)-\cot(B3-B2) \tag{11}$$

So, this plot gives the direction of the target relative movement with respect to the submarine, using only measurements of time and bearing.

In addition and refering to Figure IV, if *t0*, *B0* and *R0* are time, bearing and range at the Closest Point of Approach (CPA), then at any time *tj*,

$$\tan(Bj-B0)=\frac{TgtRelS*(tj-t0)}{R0} \quad (12)$$

and

$$R0=Rj*\sin(RelBeta) \quad (13)$$

Where *TgtRelS* is relative speed of the target with respect to the submarine and *Rj* is range at time *tj*. Thus, for a given relative movement, there are an infinite number of possible solutions, since there are three unknowns (*Rj*, *R0* and *TgtRelS*) for two equations. But in all cases *R0* and *TgtRelS*

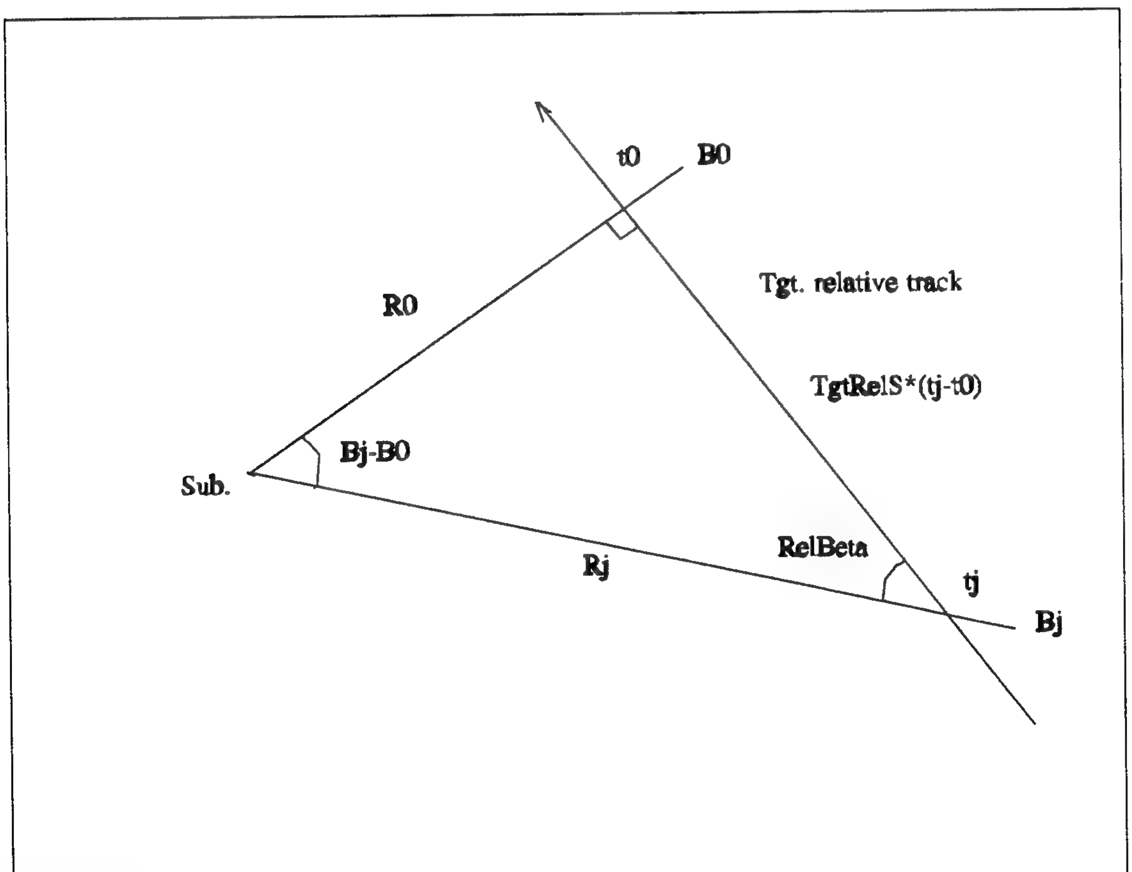

Figure IV. Relative Motion Solution

are in a constant proportion and, if a range Rj is obtained at a certain time by Ekelund ranging or any other means, equations (9) and (8) can be solved for $R0$ and $TgtRelS$ respectively. Then, solving for target course and speed from submarine and relative courses and speeds, a complete TMA solution can be achieved.

2. Implementation

Since a complete leg lasts only 10.5 minutes, both angles $(B3-B2)$ and $(B3-B1)$, are necessarily small. This results in a high sensitivity of $RelBeta$ to bearing errors. For this reason two different methods are used to compute those angles in the simulation.

The first method uses a moving average technique, obtaining estimates $AverB1$, $AverB2$ and $AverB3$ of $B1$, $B2$, $B3$ as the average of seven consecutive raw bearings. A total of 21 consecutive bearings are processed, in batches of 7, to compute the three averaged bearings (corresponding to minutes 1.5, 5.0 and 8.5 from the time of the first bearing). An estimate $TgtRelC1$ of the direction of the target's relative motion is:

$$TgtRelC1=RelBeta1+180+AverB3 \tag{14}$$

The second method uses the same bearing rates, obtained from the time-bearing plot, that are used in the Ekelund procedure. But in this case we use the first and second half of the same leg, resulting in a total of 22

bearings being processed. For a given interval of time, bearing rate multiplied by the length of the interval is an estimate of the variation of the bearing over that interval. In order to estimate the same angles and bearings that the first method does, both bearing rates are multiplied by 3.5 minutes: *(3.5*Brate1)*, *(3.5*Brate1+3.5*Brate2)* and *(MeanB2+0.5*Brate2)* are estimates of *(B3-B2)*, *(B3-B1)* and *B3* respectively, where *MeanB2* is the mean bearing of the second regression line (corresponding to minute 8.0 from the begining of the leg). A second estimate of the direction of the target's relative motion is:

$$TgtRelC2 = RelBeta2 + 180 + MeanB2 + 0.5 * Brate2 \qquad \mathbf{(15)}$$

So for each leg there are two solutions for target relative course. When an Ekelund procedure is completed, using the two solutions corresponding to the leg that contains *Tstar*, two solutions for target course and speed are obtained and stored in two-dimension arrays, indexed by turn and solution number.

C. GEOGRAPHIC PLOT

1. Description

This is a multipurpose plot that is performed on the dead-reckoning tracer (DRT); this section is focussed only on its TMA application.

The strip plot is performed primarily by recording the submarine positions at fixed intervals of time, and the corresponding smoothed bearings to the target. Then, using transparent strips, each of them keyed to a target speed, the operator attempts to find the best fit between the bearing lines and the strips. The TMA solution corresponds to that fit. This method was not implemented.

A simple method also performed on this plot is the estimation of target course and speed when two Ekelund solutions are obtained and the interval of time between them is big enough, as it is in a lag-lead-lag maneuver (see Figure V). Plotting both target positions on the DRT, course and speed can be directly measured.

The geographic plot is also used for fine tuning a TMA. For example: if an Ekelund solution is obtained and the corresponding target course and speed computed from the relative plot, the target geographical position is plotted on the DRT, then, using that position as an anchor point, the strip keyed to the computed target speed is used trying to obtain a good fit to verify the solution or make a "fine tuning" to improve it.

2. Implementation

The strip plot described was not implemented, but the simple method to estimate target course and speed from two Ekelund solutions is used in a lag-lead-lag situation. In

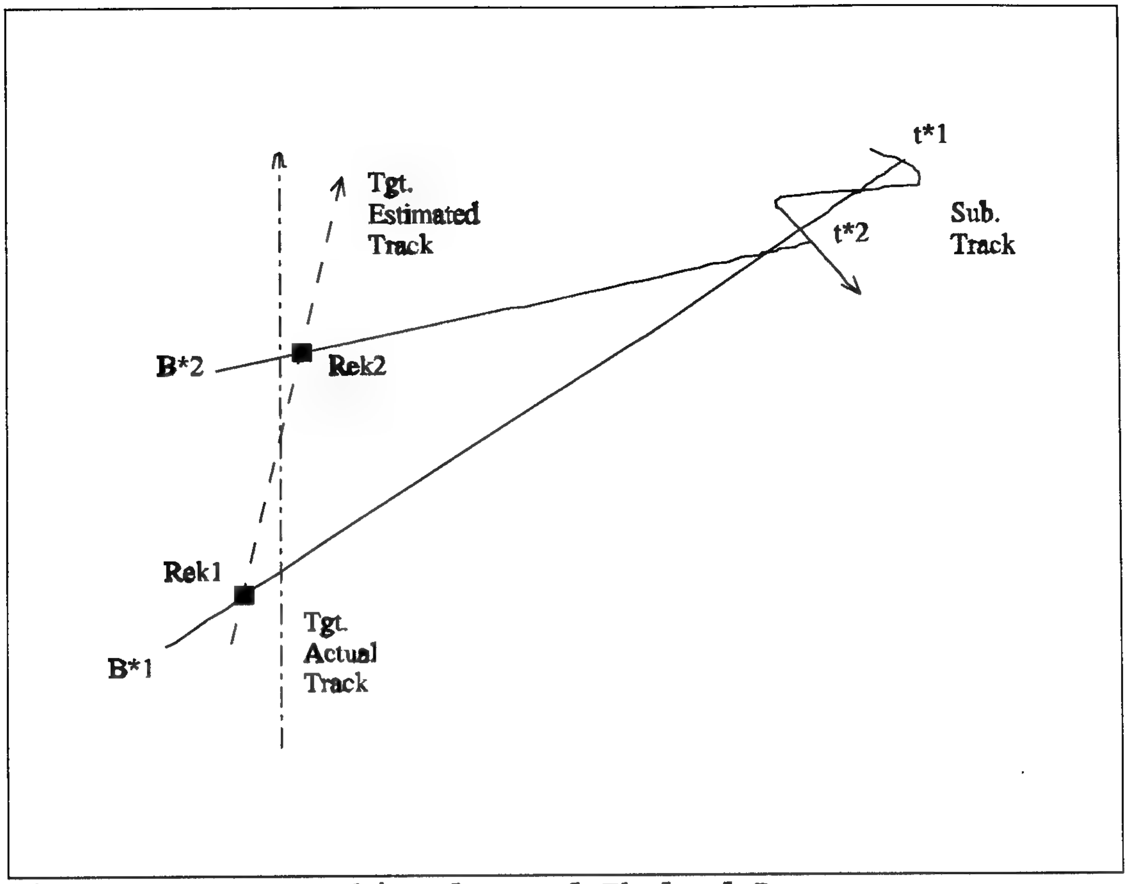

Figure V. Geographic Plot and Ekelund Ranges

that case, *Tstar* corresponding to the first Ekelund range falls on the initial leg. Since the leg lasts only five minutes, target relative course cannot be computed for that leg, and target course and speed are setimated using this alternative method.

The fine tuning method described was also implemented, using the stored data: submarine position, target bearings and times, Ekelund solutions, and relative motion plot solutions. Since the TMA procedure is composed of three legs, two Ekelund solutions and two relative motion solutions for each of them are obtained. So, there is a total of four complete TMA solutions.

A weighted average technique is used to obtain the "best solution": a weight of zero or one is assigned to each solution and the averaged data are used to forecast the current target bearing ; the error between this and the actual current bearing is computed. The averaged solution obtained with the set of weights that produces the minimum error is selected as the "best solution".

Finally, target averaged course and speed are modified in the minimum necessary amount to cancel the error, as is represented in Figure VI. For clarity purposes, only two complete TMA solutions are included, as corresponds, for

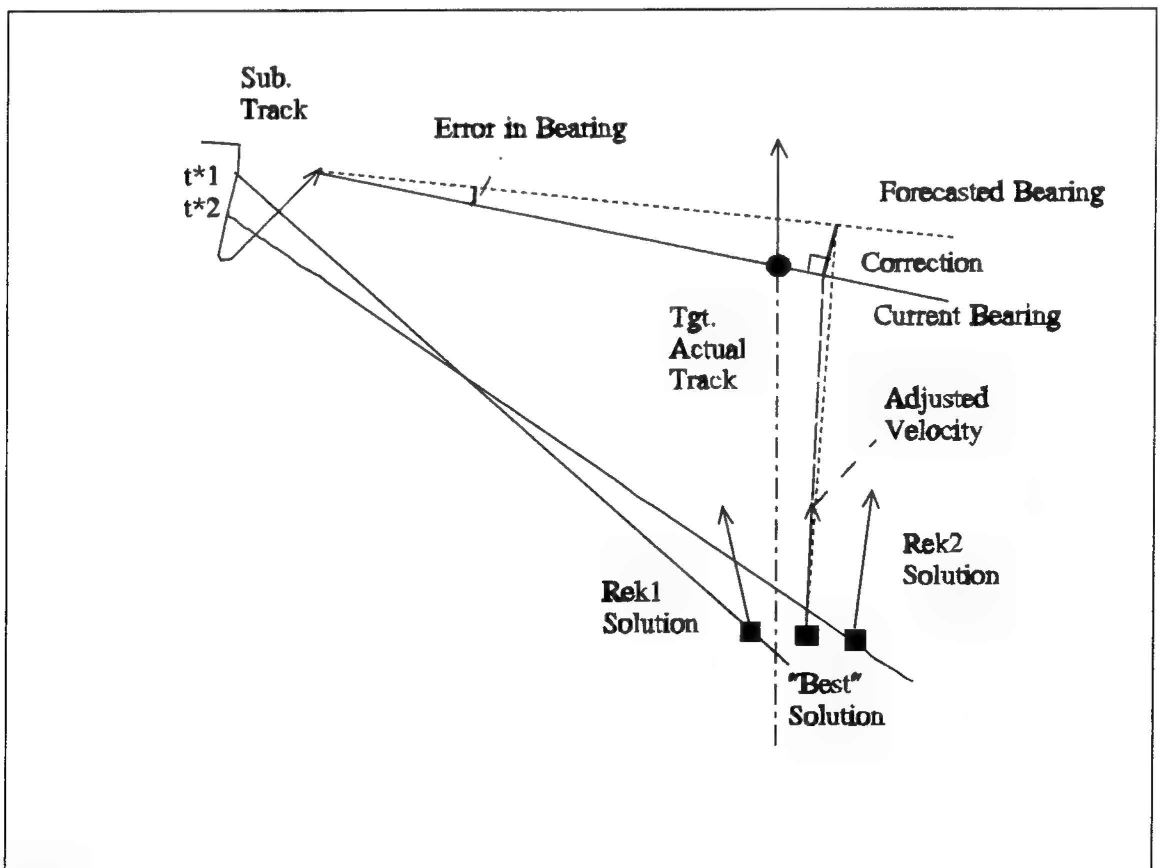

Figure VI. Construction for Adjusted Solution

example, if the best set of weights is (1, 0, 1, 0) when applying the weighted average technique to the total of four complete solutions.

There are two reasons for this complex method to be implemented. One is that the results obtained are better than those obtained by simply averaging the four solutions (all weights 1). The second is that it also reflects what is done in reality, the Commanding Officer selects the solution he considers best based on the information available and his experience.

III SIMULATION MODEL

A. DIESEL SUBMARINE MODEL

The submarine modeled does not correspond to any specific actual type, but its performance may be considered credible for a diesel submarine of modern design and construction. The main characteristics modeled are:

1. Speed and Battery

The initial battery charge level is uniformly distributed between 73% and 85%. These values are considered plausible to conduct a patrol against surface targets.

The selection of the speed to conduct the approach and attack is made as a function of the initial battery charge, according to TABLE 1.

TABLE 1
BATTERY / SPEED

Battery:	Speed:
73% - 76%	8.0 kts.
76% - 79%	10.0 kts.
79% - 82%	12.0 kts.
82% - 85%	14.0 kts.

The battery characteristics to be modeled are rate of discharge and minimum charge needed, as functions of the speed.

The percent of battery discharged per minute is assumed to increase exponentially with speed:

$$Discharge(\%/minute) = \frac{e^{Speed(kts.)/5}}{40} \qquad (16)$$

For a given rate of discharge, the battery must be over a certain charge level, otherwise that rate cannot be maintained. The minimum percent of battery required for a given speed is considered increasing proportionally to the square of the speed:

$$Minimum(\%) = (Speed(kts.)/2.5)^2 \qquad (17)$$

If this minimum value corresponding to the current speed is reached, the submarine must decrease speed. In the program, if this happens, the speed is decreased by 2.0 knots.

2. Course and Speed Changes

Submarine initial course, for each replication, is generated uniformly distributed between 060.0 and 120.0. Initial speed is always 4.0 knots.

When the submarine changes course or speed, there is a delay of 30.0 seconds until the change starts. After that delay, the course changes from the current to the ordered course at an angular speed proportional to submarine speed:

$$AngularSpeed(radians/minute) = Speed(kts.)/6 \qquad (18)$$

Similarly, the speed changes with a constant acceleration or deceleration of 2.0 knots per minute.

3. Main Sensor performance

The submarine's main sensor is a hull-mounted circular hydrophone array, with a detection range that is a function of target's speed, in accordance with TABLE 2.

TABLE 2
INITIAL RANGE / TARGET SPEED

Range:	Speed:
11.25 - 14.45 n.m.	16.0 kts.
14.45 - 18.05 n.m.	18.0 kts.
18.05 - 22.05 n.m.	20.0 kts.
22.05 - 26.45 n.m.	22.0 kts.

The measurement error for the received bearings is considered normally distributed with mean 0.0 and standard deviation of 0.5 degrees. The interval between consecutive bearings is 30 seconds.

4. Weapons performances

The submarine is armed with passive acoustic torpedoes, with a maximum range of 7.5 nautical miles and 45.0 knots of speed, which gives a maximum running time of 6.0 minutes.

The acquisition range of the torpedo's acoustic detector is proportional to the target speed squared:

$$AcquisitionRange(n.m.) = 0.0008 * TgtSpeed(kts.)^2 \qquad (19)$$

If the CPA between torpedo and target is less than this distance, the target is considered acquired by the torpedo and the submarine scores a hit.

5. Initial Position

Submarine initial position is always the origin of coordinates.

B. TARGET MODEL

1. Course and Speed

Target course is always 000.0 through the whole replication.

Target speed is chosen as a function of the initial range, according to TABLE 2. It is also constant through the whole replication.

2. Initial Position

Detection range for each replication is generated uniformly distributed between 11.25 and 26.45 n.m., this same value, but negative, is target initial ordinate *Y*.

Target initial abscissa *X*, also in n.m., is generated uniformly distributed between -24.0 and 24.0.

In that position, the target obviously has not been detected yet. The target is moved following its relative motion until it closes to the detection range. At that point, if the submarine is inside the submerged approach region, defined in Section 8.5 of reference [3], the simulated approach and attack starts. If the submarine is outside the submerged approach region the whole set of initial conditions is discarded and regenerated.

As a result of this procedure, target initial range is the detection range and initial bearing is computed as function of the starting point coordinates.

C. IMPLEMENTATION OF SUBMARINE TACTICS

1. TMA Maneuver

The TMA is always conducted through a maneuver composed of three legs: an initial short leg of 5 minutes and two complete legs of 10.5 minutes. The submarine maintains its initial speed of 4.0 knots during the initial leg, increasing to the selected approach speed during the first turn and completing the rest of the approach and attack at this speed.

The parameters that determine the difference in tactics are the courses selected for each leg. Depending on the course selected, there are three possibilities:

- Lead leg: both submarine and target speeds across the line of sight have the same sign.
- Lag leg: submarine and target speeds across the line of sight have opposite sign.
- Point leg: submarine course and target bearing are the same, submarine speed across the line of sight is zero.

A first choice is the course selected by the submarine for the initial leg. Three cases are considered:

- Tactic 1: point to the target.
- Tactic 2: maintain initial course.

- Tactic 3: change course to the initial bearing minus 70 degrees.

In Tactic 1, since the initial leg is a point, the TMA maneuver is always essentially the same, point-lead-lag. The lead angle is always 70 degrees, but the lag angle may be 00, 15 or 30 degrees, to simulate three possible variations of the tactics.

In Tactic 2, since the initial course is roughly perpendicular to the target course, the initial leg may be either a lead or a lag and, consequently, the TMA maneuver may be lead-lag-lead or lag-lead-lag. The lead angle is always 70 degrees, except for the initial leg, and the lag angle may be 00, 15 or 30 degrees.

In Tactic 3, the initial leg may also be a lead or a lag, and the TMA maneuver is the same as in Case 2.

2. Course Selection for each Leg

Depending on the tactics that are being simulated in the run, course selection for each leg is radically different.

a. Tactic 1

The submarine changes course to match the initial target bearing, so the first leg is a point. The second leg is always a lead, with lead angle 70 degrees, and the third leg is a lag, with the lag angle chosen for the run (0, 15 or 30 degrees).

b. Tactic 2

The initial course is maintained during the first leg. As a result, this leg may be a lead or a lag, depending on initial conditions.

Since, in both cases, submarine speed across the line of sight is always left (negative), if bearing rate is negative or zero, as in Figure VII-a, this means that target speed across the line of sight is also left (negative) and greater than or equal to submarine speed across the line of

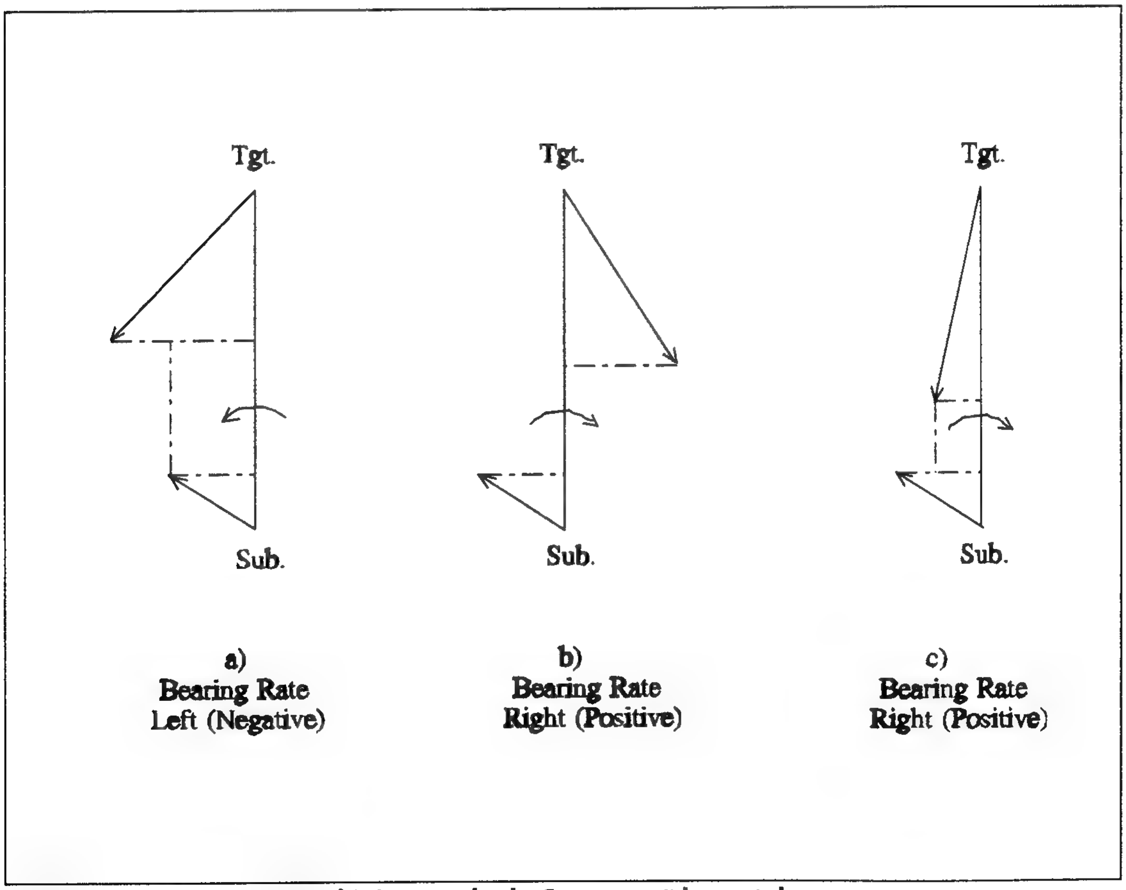

Figure VII. Possible Initial Leg Situations

sight. Thus, first leg is a lead. Second leg will be a lag,with the chosen lag angle, and third leg a second lead.

If bearing rate is positive, target speed across the line of sight may be right (positive), as in Figure VII-b, or left (negative) but less than submarine speed across the line of sight, as in Figure VII-c. There is no certainty about whether first leg is a lag or a lead; the worst situation, a lag, is assumed and the submarine selects a presumably lead angle for the second leg. After this leg, the situation is clarified: if second leg was a lead, the chosen lag angle is selected for third leg, other wise a lead angle is selected.

c. Tactic 3

The submarine changes course to the initial bearing minus 70 degrees. After that the situation is the same as in Tactic 2.

3. Final Attack Decision

Once the TMA maneuver is completed, the submarine tries to obtain the best possible firing position. Depending on its position with respect to target, there are three possible decisions.

a. Launch Immediately

The submarine will launch immediately if, in accordance with its TMA solution, it is within half of the maximum torpedo range, or if it is within maximum torpedo range and is unable to reach a closer firing position.

b. Close the Target and Launch

The submarine will close the target if, in accordance with its TMA solution, it is outside of maximum torpedo range or outside a half of maximum torpedo range and, in both cases, is able to obtain a better firing position. In no circumstances will the submarine intentionally approach the target closer than half of maximum torpedo range.

c. Abandon the Attack

The submarine will abandon the attack if, in accordance with its TMA solution, it is outside maximum torpedo range and is unable to close the target and reach a firing position. In other words, the submarine finds itself outside the submerged approach region.

D. MEASURES OF EFFECTIVENESS

At the end of the attack, depending on the results, each replication is included in one of the following possible categories:

- Category 0: after its TMA maneuver, the submarine is outside the submerged approach region but, because of a poor solution, does not even know it and proceeds with the attack.
- Category 1: after its TMA maneuver, the submarine is outside the submerged approach region and, because of a better solution, knows it and abandons the attack.
- Category 2: after its TMA maneuver, the submarine is inside the submerged approach region but, because of a poor solution, believes that is outside and abandons the attack, missing an opportunity.

- Category 3: the submarine reaches a torpedo release point and launches a torpedo that, because of a poor TMA solution, misses the target.
- Category 4: the submarine reaches a torpedo release point and launches a torpedo that acquires and hits the target.

The number of replications included in Category 4, in other words, the number of successful attacks on each simulation run is used as the primary Measure of Effectiveness (MOE) of the particular tactical variation employed in that run.

E. SAMPLE RUN

To validate the model, a single individual replication was performed, with the following parameters:

- Initial Battery = 78%.
- Detection Range = 16.0 n.m.
- Target initial abscissa = 7.0 n.m.

The tactical variation selected was: change course to initial bearing minus 70 degrees and lag angle 30 degrees (Tactic 3, Variation 3).

All data and intermediate results for each iteration as well as the final results from the different computational methods implemented were stored in a file for its study and validation.

1. Diesel Submarine Model

In accordance with initial battery charge of 78% and Table 1, approach speed selected was 10.0 kts.

The rate of discharge at 10 kts. according to equation (16), multiplied by the time this speed was maintained (from minute 9.5 to 32) was equal to the battery discharged in that time (from 77.4% to 73.2%):

$$\frac{e^{10/4}}{40}*(32-9.5)=4.2=77.4-73.2 \qquad (20)$$

At minute 6.5 the speed was increased from 4.0 to 10.0 kts. At the rate of 2 kts. per minute, the change started at minute 7.0, after the 30 sec. delay, and ended at 10.0.

At minute 19.5 the course was changed from 183.7 to 067.3. The angular speed, according to equation (18) was:

$$Angularspeed=\frac{10}{6}rad./\text{min.}=95.5°/\text{min.} \qquad (21)$$

The change started at minute 20.0, after a 30 sec. delay, and ended at minute 21.5.

2. Target Model

In accordance with detection range of 16.0 n.m. and TABLE 2, speed selected was 18.0 kts.

With target initial abscissa of 7.0 n.m., and according to relative motion and detection range, approach and attack started with target position 155.4 and 16.0 n.m. from

submarine at minute 0.0, since with these data the submarine is inside the submerged approach region.

3. TMA Procedures

All necessary data for TMA procedures are obtained from simulated Time-Bearing plot through least squares regression.

a. *Time-Corrected Ekelund Range*

Ekelund range is computed according to equation (1). For example, in the second ranging maneuver, *Brate1 = -1.8, Brate2 = -0.4, t1 = 17.5, t2 = 24.0, B1 = 140.9, B2 = 133.4, SSalos1 = 6.8, SSalos2 = -9.1:*

$$Rek=\frac{180}{60\pi}\frac{(-9.1-6.8)}{(-1.8+0.4)}=10.9 \qquad (22)$$

and according to equation (2):

$$Tstar=\frac{17.5*(-1.8)-24*(-0.4)+140.9-133.4}{-1.8+0.4}=10.4 \qquad (23)$$

Corresponding bearing at this time: *Bstar = 150.7.*

b. *Relative Motion Plot*

Target relative course is computed according to equation (11) and using the two different methods, averaged bearings and linear regression, to obtain both angles *(B3-B1)* and *(B3-B2)*. In leg two, for example, averaged bearings are *B1 = 150.3, B2 = 145.7, B3 = 140.1* and

$$RelBeta2 = \cot^{-1}[\cot(140.1-145.7)-2\cot(140.1-150.3)] = 47.5 \tag{24}$$

Using linear regression, *Brate1 = -1.2, Brate2 = -1.8* and

$$RelBeta2 = \cot^{-1}(\cot[3.5(-1.8)]-2\cot[3.5(-1.8-1.2)]) = 30.0 \tag{25}$$

Adding the reciprocal of the corresponding bearings, computed relative courses are respectively *010.8* and *351.4*.

c. Geographic Plot

In this run, only the fine tuning method was performed. Corresponding data for each solution are the following:

- Ekelund solution 1: *Rek1 = 9.120 n.m., Tstar1 = 19.1, Bstar1 = 140.5* and the two associated velocity vectors, *TgtC1 = 020.7, TgtS1 = 7.2 kts., TgtC3 = 341.6, TgtS3 = 12.4 kts.*
- Ekelund solution 2: *Rek2 = 10.937 n.m., Tstar2 =10.4, Bstar2 = 150.7* and the two associated velocity vectors, *TgtC2 = 015.1, TgtS2 = 16.4 Kts., TgtC4 = 347.4, TgtS4 = 29.9 Kts.*

In this particular case, the set of weights that gives the best averaged solution is (0, 1, 1, 1) with forecasted bearing 126.1, only 2.1 degrees apart of actual bearing 128.2. Adjusted solution at minute 32 is bearing 128.2, range 3.881 n.m., target course 354.6, target speed 18.6. Figure VIII shows a graphical representation of this plot.

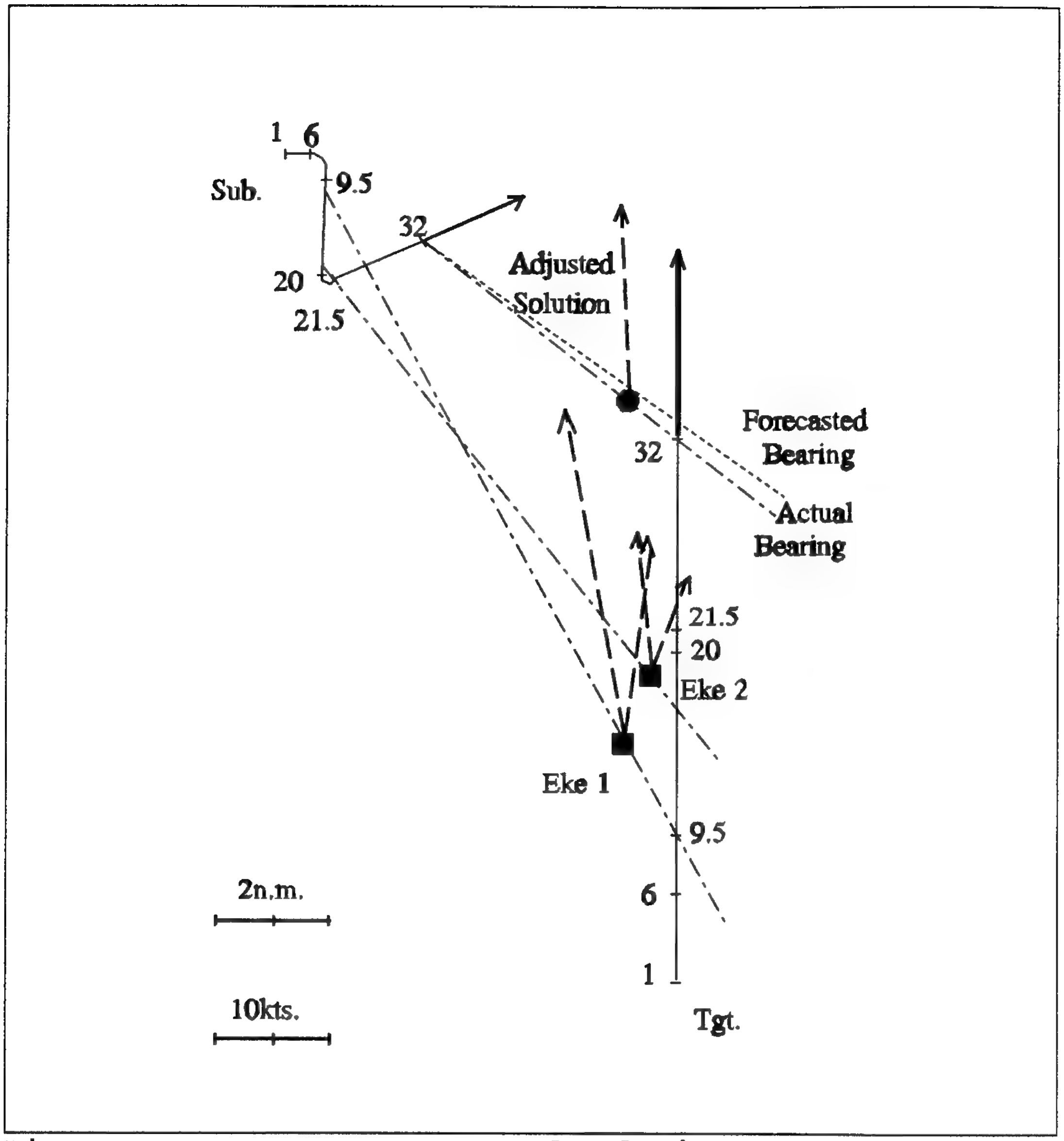

Figure VIII. TMA Maneuver and Solutions

4. Implementation of Submarine Tactics

a. Course Selection for each Leg

According to the selected tactic, as soon as the target was detected, the course was changed to the initial bearing minus 70 degrees:

$$Course = 155.4 - 70 + B_e = 085.4 \tag{26}$$

where *Be* is measurement error.

In the first leg, submarine speed across the line of sight was left (negative), bearing rate was also left (negative), so target speed across the line of sight was left (negative) as well, and first leg was a lead. Second leg would be a lag and course equal bearing plus lag angle (30 degrees):

$$Course = 153.9 + 30 + B_e = 183.7 \tag{27}$$

Third leg would be a lead and corresponding course equal bearing minus 70 degrees:

$$Course = 137.2 - 70 + B_e = 67.3 \tag{28}$$

b. Final Attack Decision

Adjusted solution data are used to verify that submarine is inside the submerged approach region and compute torpedo course and running time:

$$TorpC = 128.2 - \sin^{-1}\left[\frac{18.6}{45}\sin(128.2 - 354.6)\right] = 110.8 \tag{29}$$

Where *TorpC* is torpedo course, and

$$TorpRT = \frac{3.881 * 60}{45\cos(128.2 - 110.8) - 18.6\cos(128.2 - 354.6)} = 4.2 \tag{30}$$

Where *TorpRT* is torpedo running time in minutes.

Since this time is less than a half of maximum running time (5.0 min.), the torpedo is launched immediately.

5. Result of the Attack

To determine the result of the attack, using torpedo course and true target data, CPA between torpedo and target is computed and compared with torpedo acquisition range, according to equation (18):

$$AcquisitionRange = 0.0008*18^2 = 0.26\,n.m. \qquad (31)$$

Solving for torpedo-target relative velocity vector gives: relative course 128.9, relative speed 54.1. These data combined with true target bearing 128.1 and distance 5.687 n.m., gives: CPA time 6.3 min. and CPA range 0.063 n.m. Since this distance was less than acquisition range, the torpedo acquired and hit the target, so the attack was a success and included in Category 4.

IV RESULTS AND CONCLUSIONS

A. EXPERIMENTAL DESIGN

The result of an approach and attack will depend, among other factors, on the tactical variation applied and the submarine's battery condition. These are the only factors under the submarine's control or, in the case of battery condition, known at the beginning of the approach. Since there are three possible tactics, with three variations for each of them and four battery conditions, a total of 36 different combinations of tactical variation and battery conditions are possible.

In order to reduce the number of runs, the experiment initially includes only strategies where the TMA tactic is independent of battery condition, a total of 9 Monte Carlo simulation runs, in 3 groups of 3, each group corresponding to one of the three tactics under study:

- Tactic 1: point to the target.
- Tactic 2: maintain initial course.
- Tactic 3: change course to the initial bearing minus 70 degrees.

Each particular run on these groups corresponds to the following tactical variations:

- Variation 1: lag angle 00 degrees.
- Variation 2: lag angle 15 degrees.
- Variation 3: lag angle 30 degrees.

These first nine runs will be used to determine which is the best tactical variation and eliminate all of those that are significantly worse. Then, for every tactical variation that is not eliminated, the effect of permitting the TMA tactics to depend on battery condition will be studied. For example, if all but two tactical variations are eliminated, two series of four runs will be necessary.

B. SIMULATION RESULTS

The results of the first nine runs are shown in TABLE 3.

TABLE 3
SIMULATION RESULTS

Category:		**0**	**1**	**2**	**3**	**4**
Tact. 1	**Var. 1**	330	723	1636	1726	5585
	Var. 2	297	795	1465	1879	5564
	Var. 3	340	873	1298	1298	5456
Tact. 2	**Var. 1**	451	527	1606	1741	5675
	Var. 2	571	656	1388	1743	5642
	Var. 3	643	781	1299	1755	5522
Tact. 3	**Var. 1**	408	523	1632	1619	5818
	Var. 2	491	669	1443	1588	5809
	Var. 3	596	834	1319	1679	5572

In accordance with these results, the maximum number of successful attacks (5,818) is achieved using Variation 1 of Tactic 3: change course to the initial bearing minus 70 degrees and select a lag angle of 0.0 degrees.

Each replication may be considered as an independent Bernouilli trial with probability of success p, and each run a Binomial experiment consisting of 10,000 trials. An estimate of mean p is the number of successes divided by the number of trials. For the best tactical variation:

$$p=\frac{5,818}{10,000}=0.5818 \tag{32}$$

and an estimate of standard deviation of this mean is:

$$s_p=\sqrt{\frac{p*(1-p)}{n-1}}=\sqrt{\frac{0.5818*(1-0.5818)}{9,999}}=0.0049 \tag{33}$$

In accordance with these values, Tactic 3 Variation 1 is significantly better than all other tactics except Tactic 3 Variation 2 (5,809 successful attacks): change course to initial bearing minus 70 degrees and select a lag angle of 15.0 degrees.

To check the effect of the submarine's battery condition, a series of 4 runs was performed for each tactical variation. In each run the submarine's battery condition and tactical variation were fixed to a particular value for 10,000 replications. The result of these 8 runs are presented in TABLE 4.

In accordance with these results, there is an interaction between battery condition and tactical variation: with low battery condition, Variation 2 (lag angle 15.0 degrees)leads to better results than Variation 1 (4,732 versus 5,008 successes), Variation 1 (lag angle 0.0 degrees) leads to better results (6,209 versus 6,095 successes) when the battery condition is high.

TABLE 4
SIMULATION RESULTS

Category:		**0**	**1**	**2**	**3**	**4**
Var. 1	**8kts.**	676	602	2155	1835	4732
	10kts.	385	493	1728	1666	5728
	12kts.	379	485	1382	1577	6177
	14kts.	375	644	1207	1565	6209
Var. 2	**8kts.**	679	629	1911	1773	5008
	10kts.	428	560	1521	1677	5814
	12kts.	451	696	1254	1513	6086
	14kts.	479	774	1170	1482	6095

An effect easily observable in TABLE 4 is that for both tactical variations, when the battery is low (approach speed 8 kts.), the number of successes is substantially smaller than in all other cases.

A secondary result obtained concerns the adequacy of the battery condition after the attack. The mean battery data for

the initial eight runs, grouped by initial battery condition is shown in TABLE 5. These data show that the battery condition at the end of the attack is in all cases more than 70%, which allows the submarine to do an evasion at maximum speed in case it is necessary.

TABLE 5
BATTERY MEAN DATA

Initial Charge	Approach Speed	Final Charge	Total Discharge
74.5	8.0	70.9	3.6
77.5	10.0	72.3	5.2
80.5	12.0	72.9	7.4
83.5	14.0	72.3	11.2

Additional information obtained is the influence of target speed on the results. The total number of replications and successful attacks and percentage of successes for the 8 final runs, corresponding to each target speed is presented in TABLE 6.

TABLE 6
EFFECT OF TARGET SPEED

Speed	Replications	Successes	Percent
16.0	13,780	8,945	64.91
18.0	17,857	11,286	63.20
20.0	21,996	12,771	58.06
22.0	26,367	12,847	48.72

Although the number of replications (i.e. the number of detections) and the number of successful attacks increase, the percentage of successes decreases with target speed.

C. CONCLUSIONS

The tactic that leads to the best result is: once the target is detected, change course to the initial bearing minus the lead angle of 70 degrees to conduct a clear lead-lag-lead or lag-lead-lag maneuver; the lag angle should be selected according to battery condition, for low battery condition a lag angle of 15 degrees produces better results, while for high battery condition this angle should be decreased to 0 degrees. Using this tactic, 58.18% of attacks are successful. In addition only 9.31% of the time does the submarine end outside the submerged approach region, when using this tactic. If the submarine conducts more radical maneuvers, the accuracy of the solutions is improved, but the number of times that the submarine ends its TMA maneuver outside the submerged approach region increases inordinately. For example, applying the same Tactics 3 but selecting a lag angle of 30 degrees, the submarine ends outside the submerged approach region 14.3% of the time and the number of successful attacks decreases to 55.72%. That indicates also that in a lag, closing the target is the most important effect.

Applying adequate tactics, a modern diesel submarine can reach a favorable attack position, closing the target within torpedo range and generating a TMA accurate enough to place the weapon within acquisition range, roughly 60% of the times, even taking into account its battery limitations.

LIST OF REFERENCES

1. J. J. Ekelund, *A Means of Passive Range Determination*, Commander Submarine Forces, Atlantic Fleet, Quarterly Information Bulletin, Summer 1958.

2. D. H. Wagner, *Naval Tactical Decision Aids*, Naval Postgraduate School, Military Operations Research Lecture Notes, September 1989.

3. B. O. Koopman, *Search and Screening*, Operations Evaluation Group, Office of the Chief of Naval Operations, Report 56, 1949.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, Virginia 22304-6145	2
2.	Library, Code 52 Naval Postgraduate School Monterey, California 93943-5002	2
3.	Prof. Alan R. Washburn, Code OR/Ws Naval Postgraduate School Monterey, California 93943-5002	1
4.	Prof. James N. Eagle, Code OR/Er Naval Postgraduate School Monterey, California 93943-5002	1
5.	Pedro F. Coll Cuartel General de la Armada (GIMO) C./ Montalban 2 Madrid, Spain	3